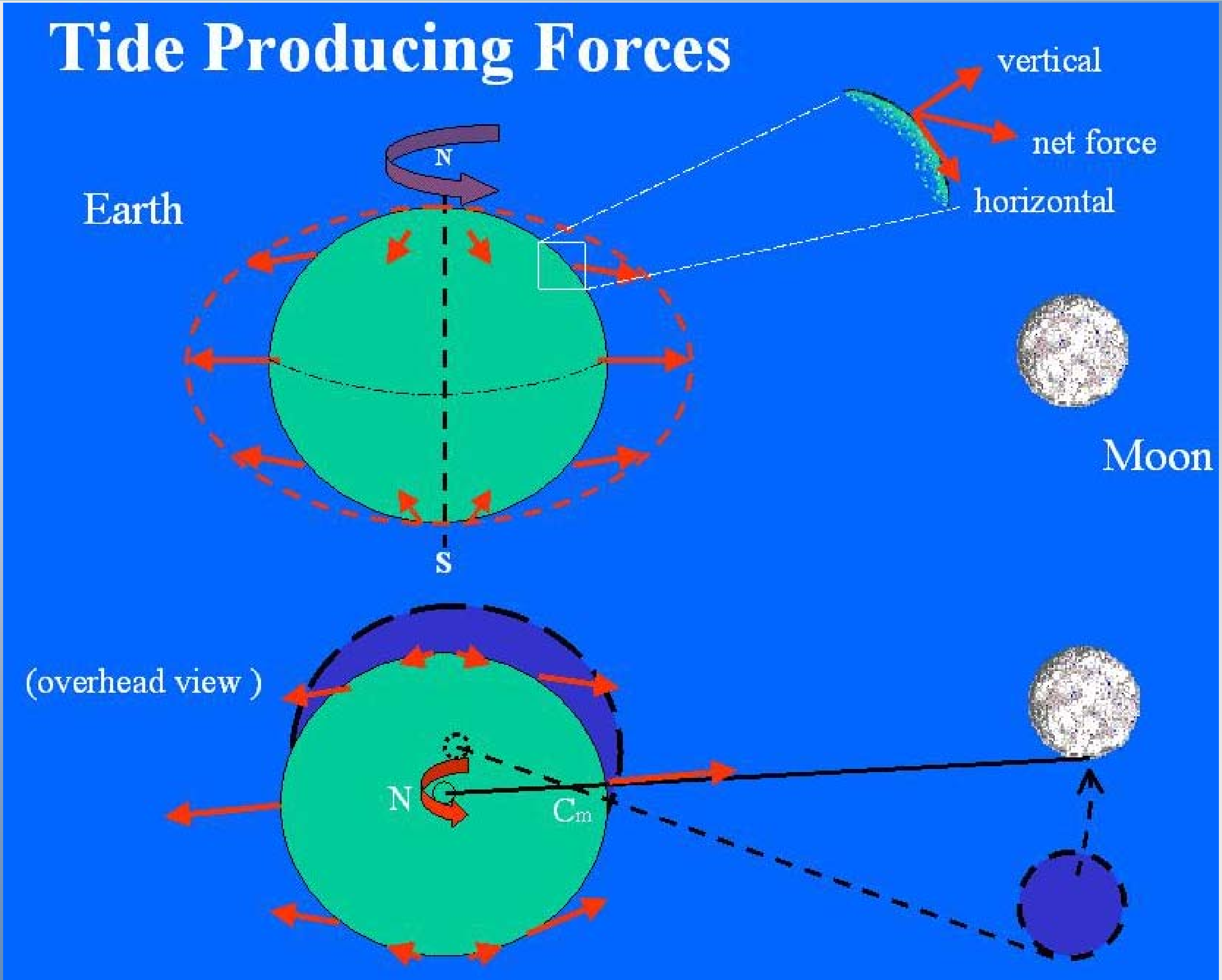
Tidal Theory
Tide Producing Forces

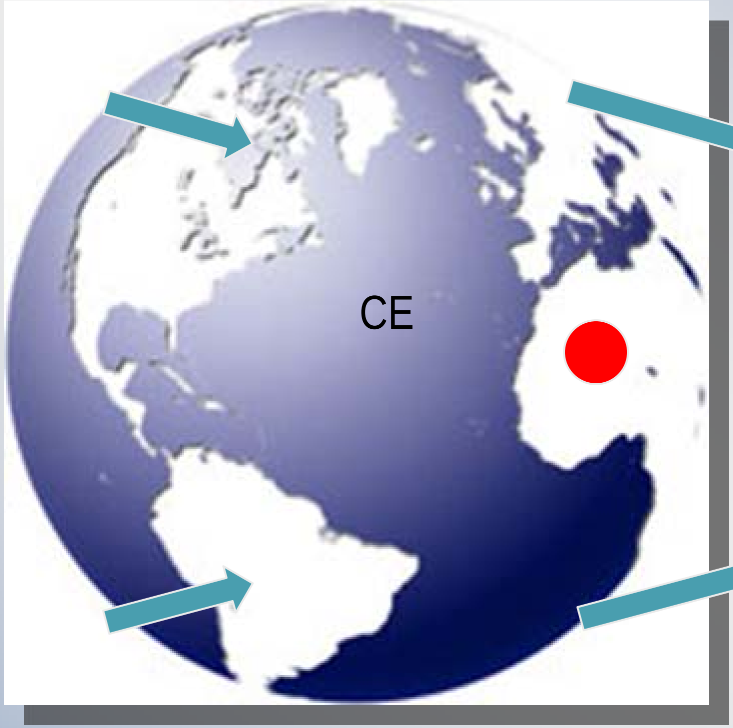
Stephen Gill
Stephen.Gill@noaa.gov

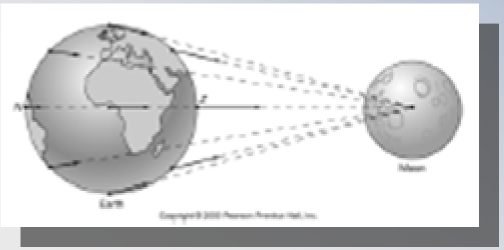


Tides –

The periodic variation in the surface level of the oceans and of bays, gulfs, inlets, and estuaries, caused by gravitational attraction of the moon and sun.

Tide Producing Forces - Gravitational Force





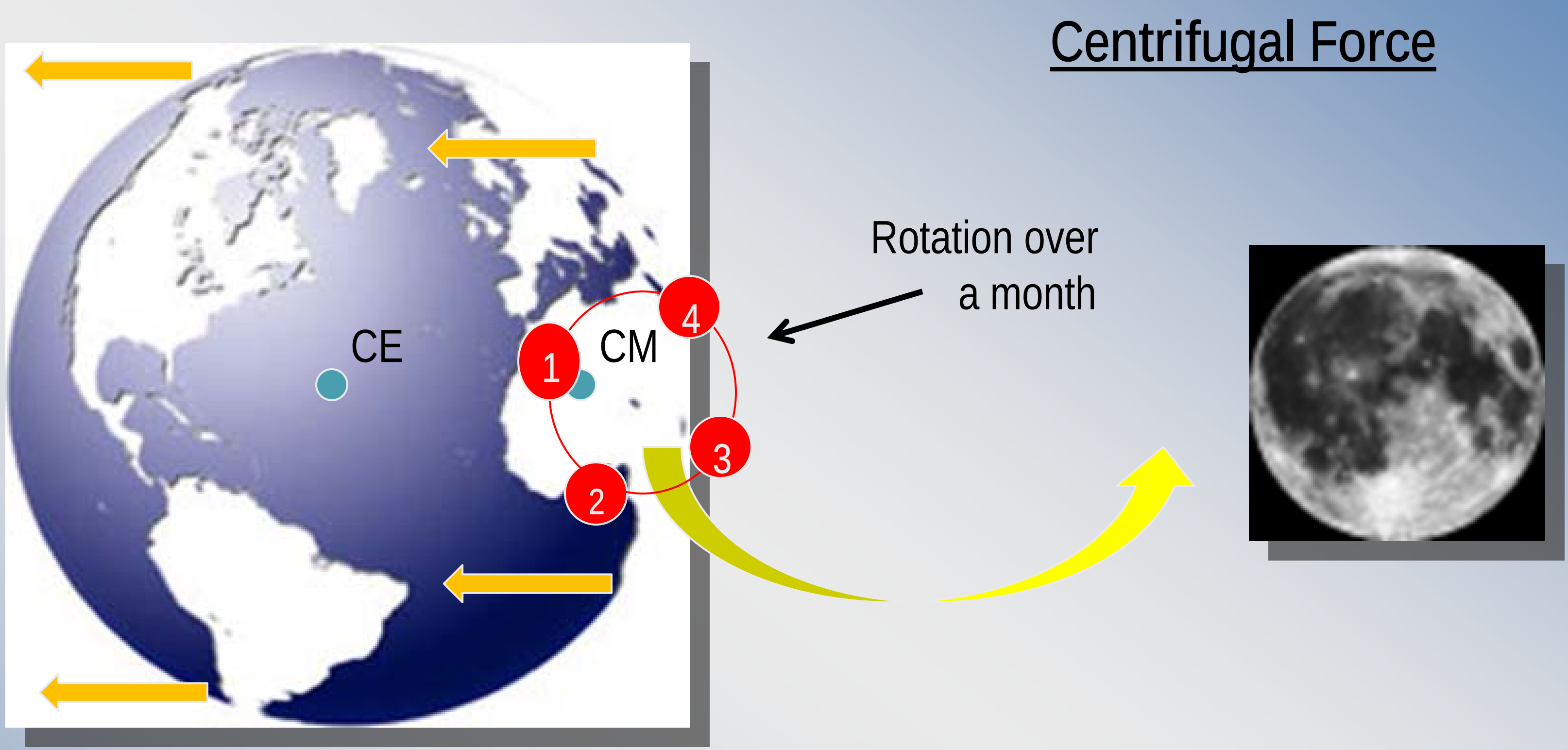
Center of Earth (CE)

Center of Mass of Earth-Moon System (CM)

Gravitational Force from Moon (and the Sun, but much smaller)

- Quantified by Newton's law: $F_g = G (M_1 \times M_2) / R^2_{radius}$
  - Larger R between Sun and Earth: $F_{g\ (Earth\text{-}Moon)} \sim 2 \times F_{g\ (Earth\text{-}Sun)}$
- Towards the center of the moon (and sun)
- Smaller for the points farther away from the moon (and sun)

Tide Producing Forces - Centrifugal Force

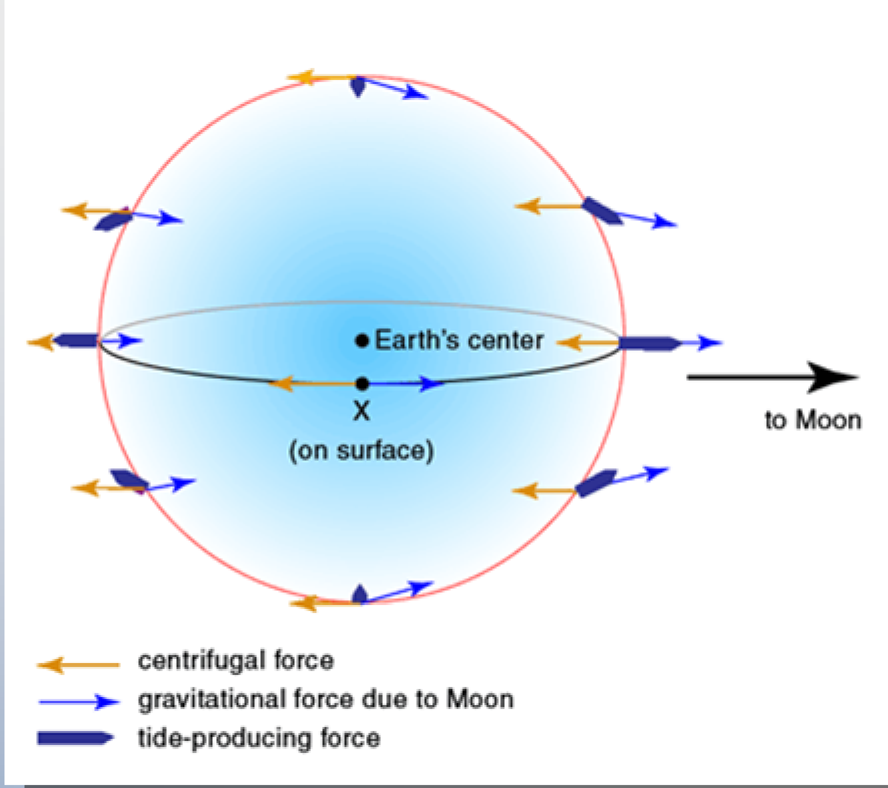


- Centrifugal force arises from the moon and earth revolving around their common center of mass (CM)

- Centrifugal force is directed away from the moon and constant in strength at every point on the Earth's surface

- Centrifugal force balances the gravitational force of the moon at the center of the earth

Tide Producing Forces - Tractive Force



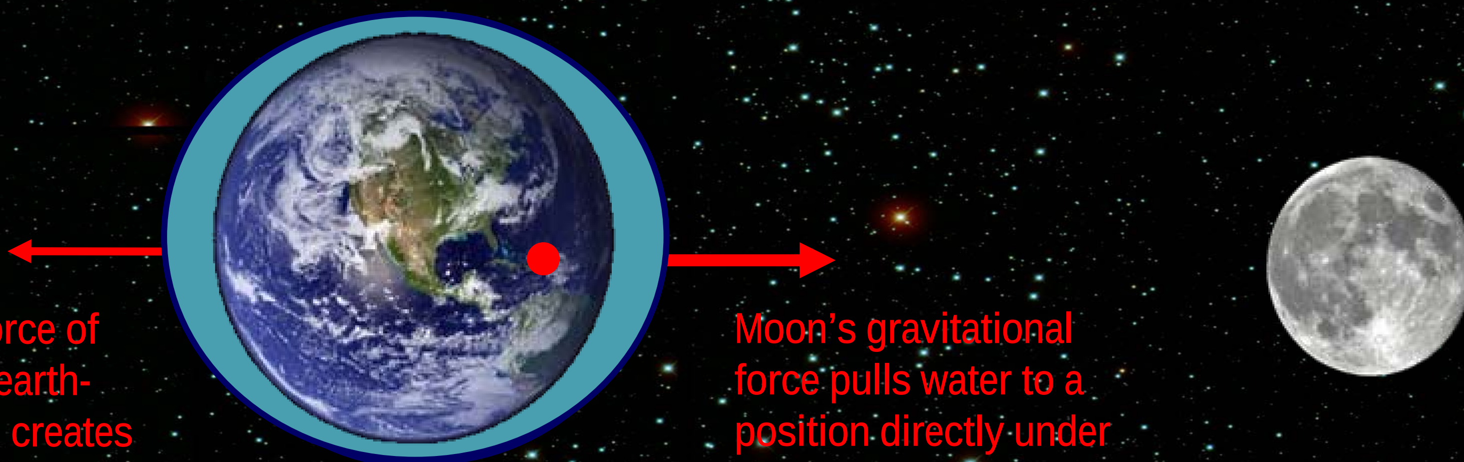
Gravitational Force: (Fg) Greater where closer to the moon

Centrifugal Force (Fc): Same everywhere on Earth

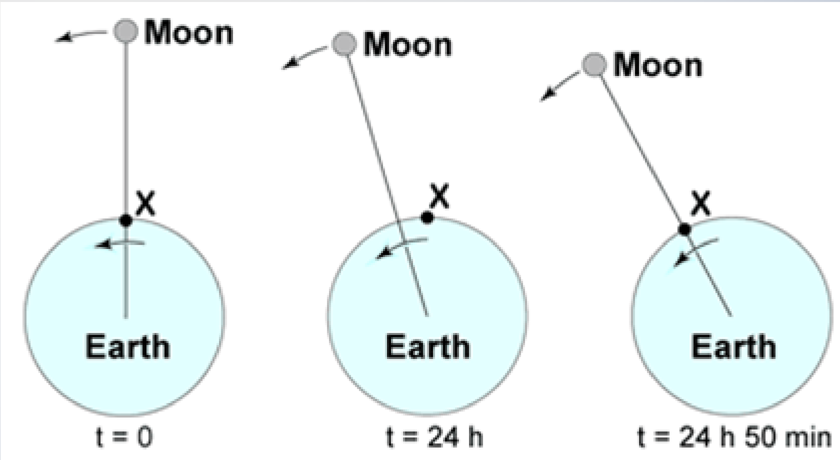
Tractive Force (Ft): Tide producing force that is the resultant horizontal component of the vector addition of Fg and Fc

Tidal Theory

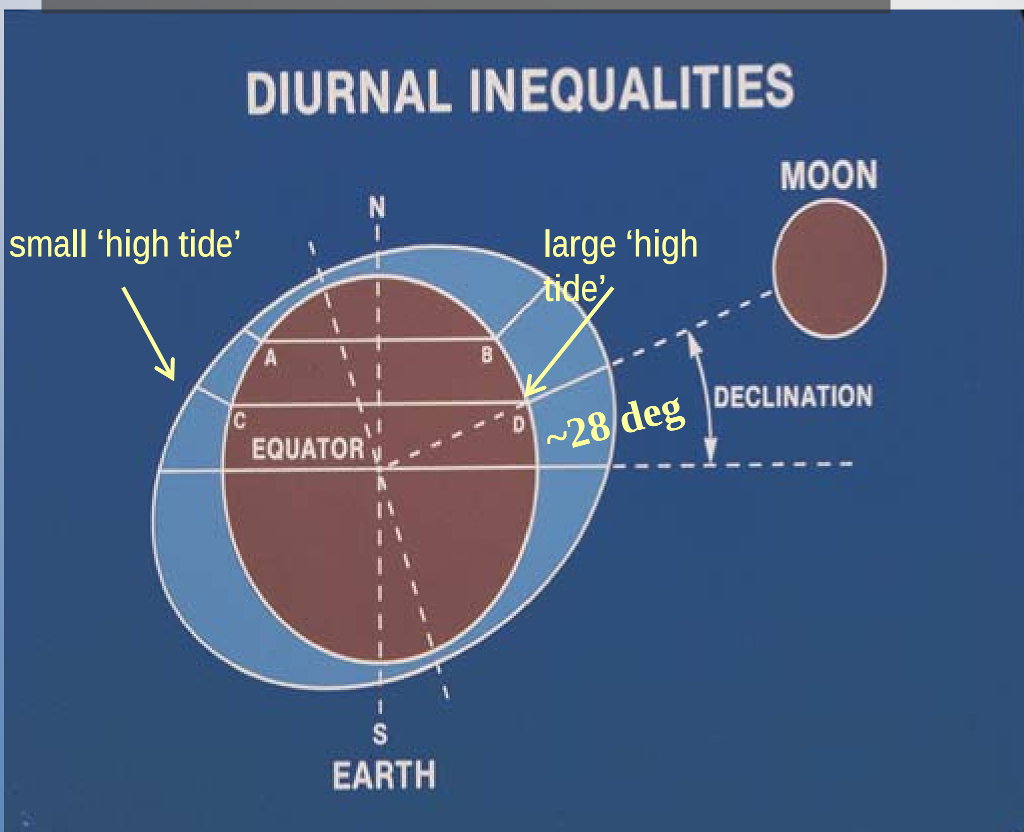
Newton – it's all about gravity (well, mostly)



Tide Producing Forces



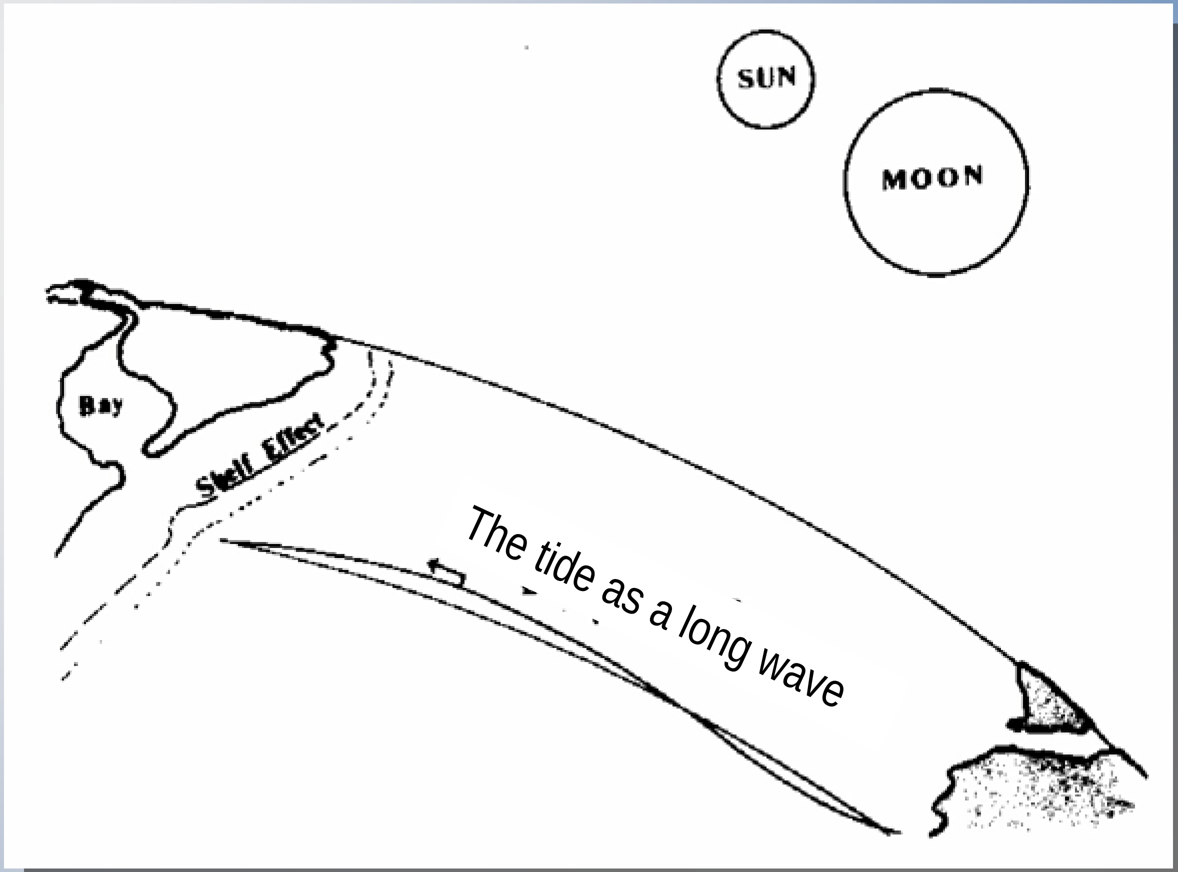
- 2 highs and 2 lows per lunar day – often referred to as semidiurnal lunar constituent, M_2
- Lunar or tidal day = 24.84 hr (Solar day = 24 hr)
  - spot on the earth has to 'catch up'



- Moon's orbit and tidal bulges are tilted relative to the earth's equator
- Diurnal Inequalities: 2 high waters not equal to each other

However, there are many other factors contributing to the observed tides

- Changing orbital parameter of the Earth-Moon-Sun system
- Land masses
- Basin size and orientation
- Coriolis force from earth's own rotation



Spring and Neap tides

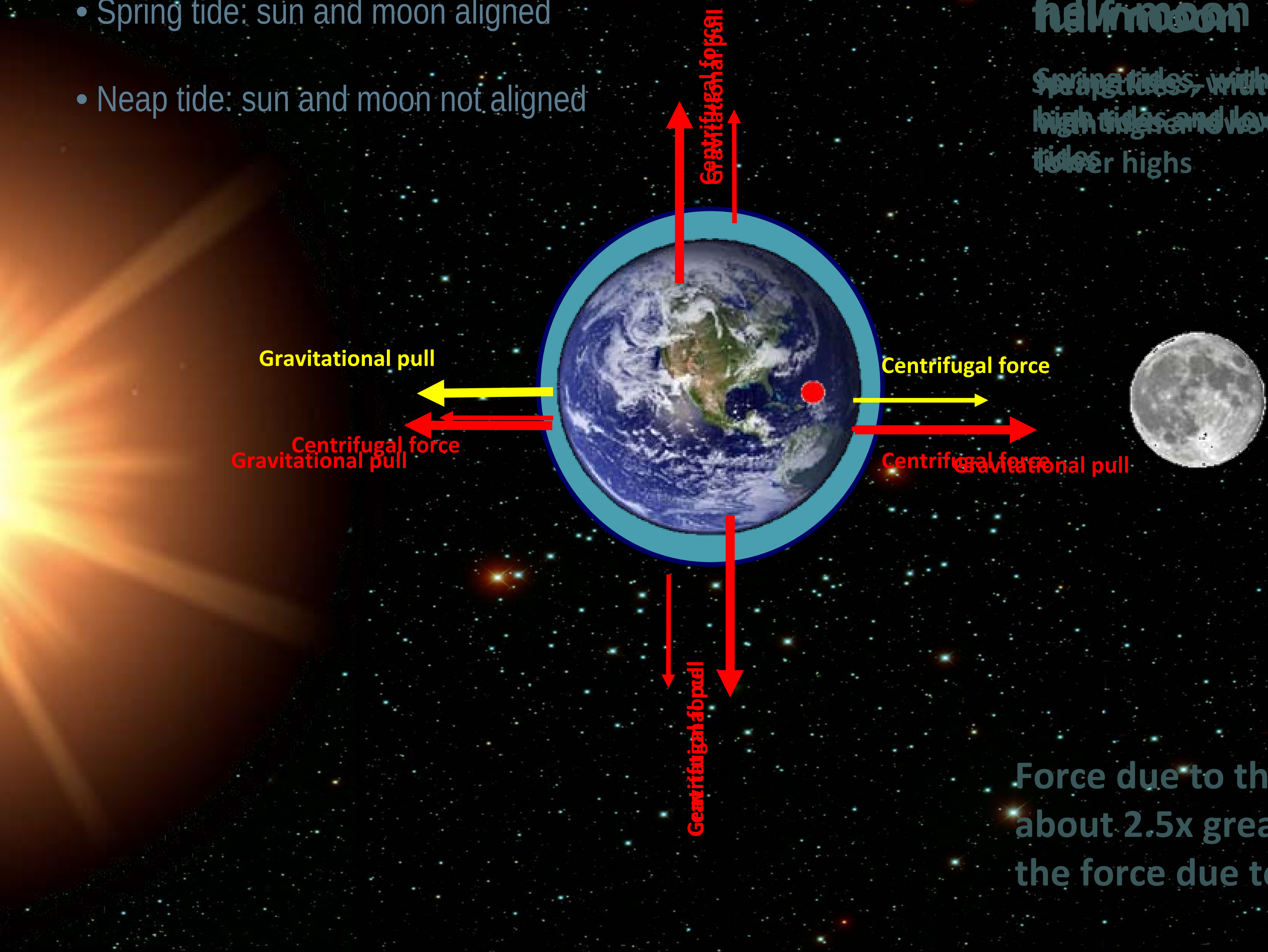
- Spring tide: sun and moon aligned
- Neap tide: sun and moon not aligned

full moon

half moon

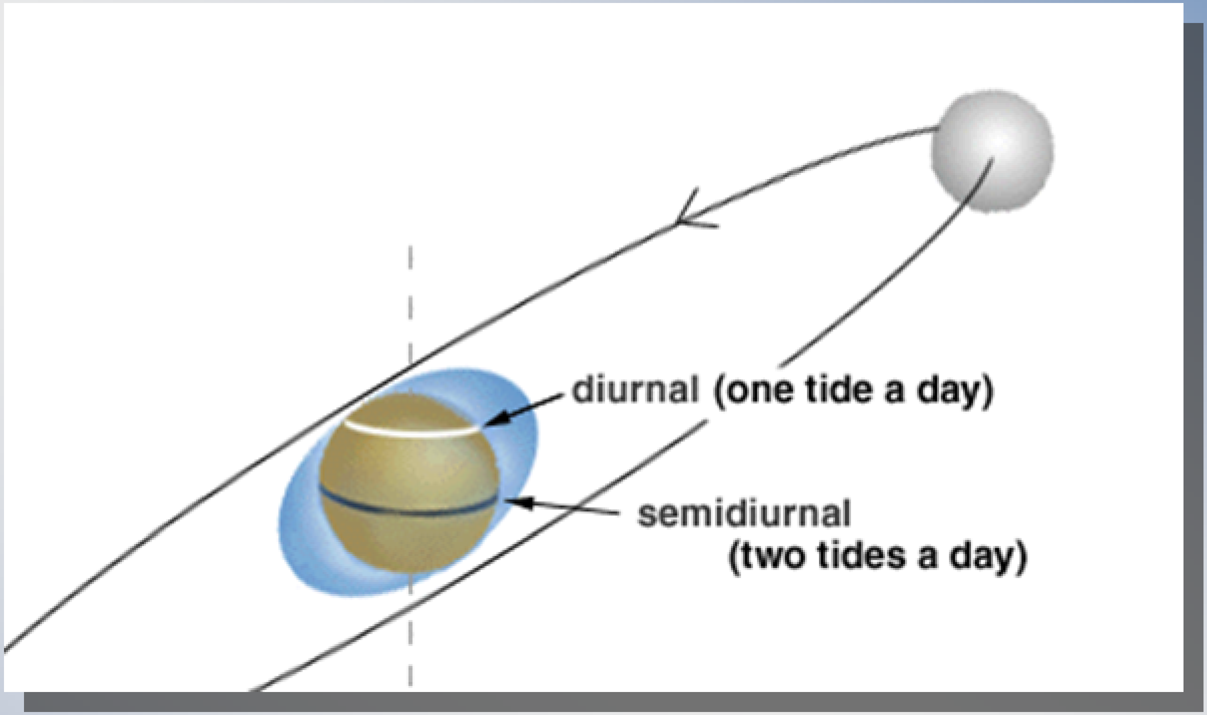
Spring tides, with higher high tides and lower low tides

Neap tides, with higher lows and lower highs



Force due to the moon is about 2.5x greater than the force due to the sun

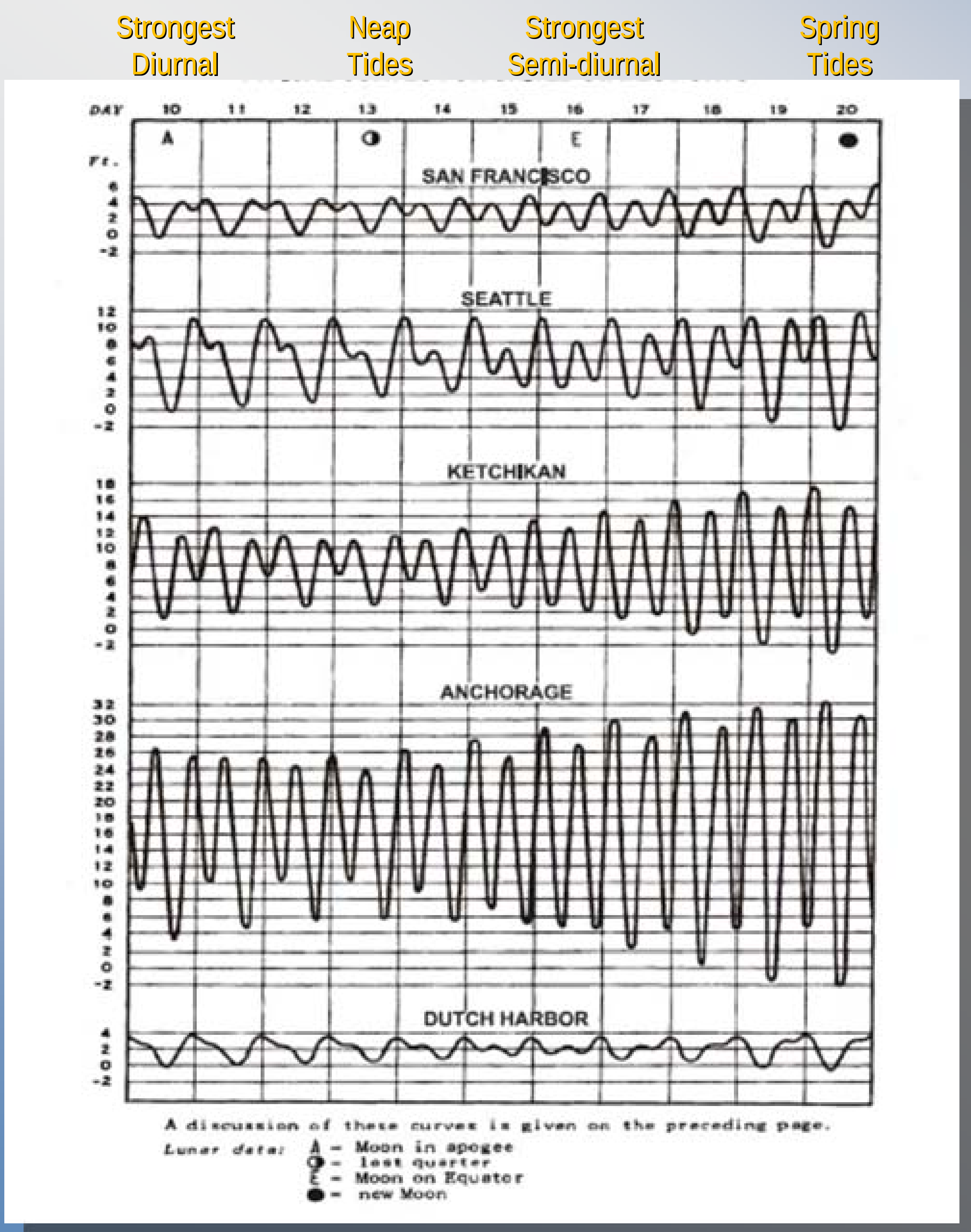
Changing Orbital Parameters



Tilt of moon's orbit to plane of the Earth's equator

- Moon will cross the equator twice in a lunar month
- Other times, moon is north or south and tidal bulges asymmetric to Earth's axis
- Diurnal inequality between highs creates diurnal frequency energy
  - Zero in strength when moon over the equator, max value when moon is farthest north or south

Changing Orbital Parameters



<u>Stations respond similarly</u>

- Greatest range during full moon (spring tide)

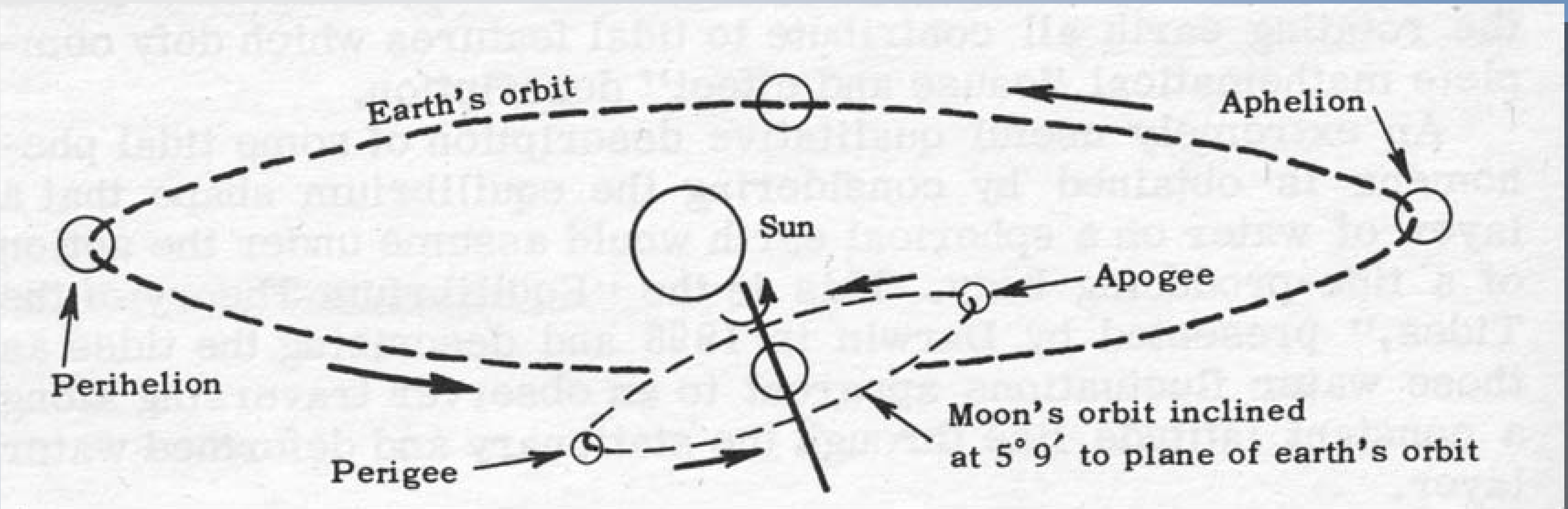
- Noticeably semi-diurnal when the moon is over the equator

- Smallest range during quarter moon (neap tide)

- More diurnal in nature when moon is in apogee and at a maximum northern/southern declination

Overall tide signal reflects their "tide type"– discussed later – from local hydrodynamic

Changing Orbital Parameters



Changes in the moon's orbit around the earth and the earth's orbit around the sun give rise to the tidal frequencies, or constituents

- Moon and Earth orbits are elliptical (affects 'R' in Newton's gravitational force equation)
- Both orbital planes are at angles to the plane of the Earth's equator
- Their ellipsoidal shapes change over time
- The orbital plane angles change over time

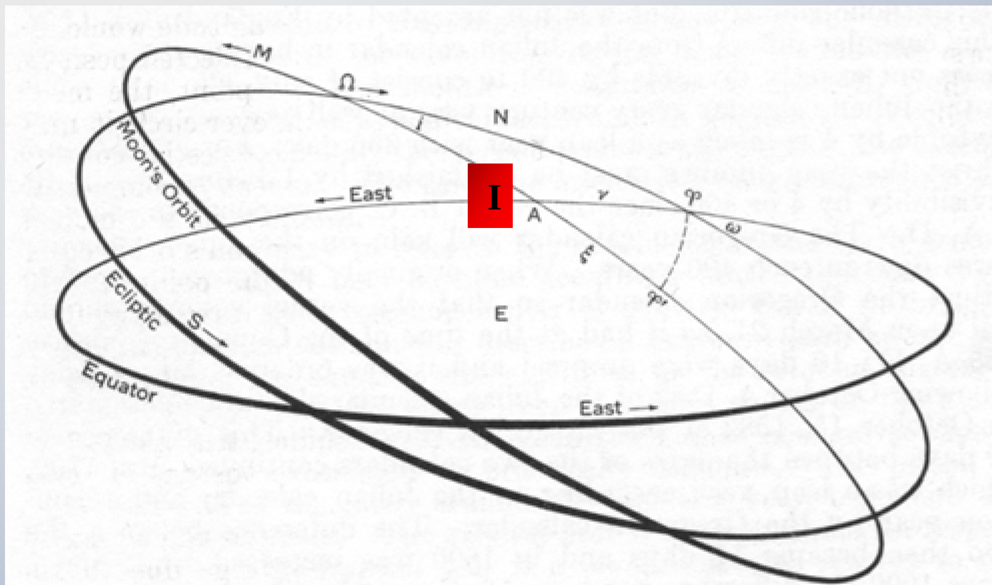
For example

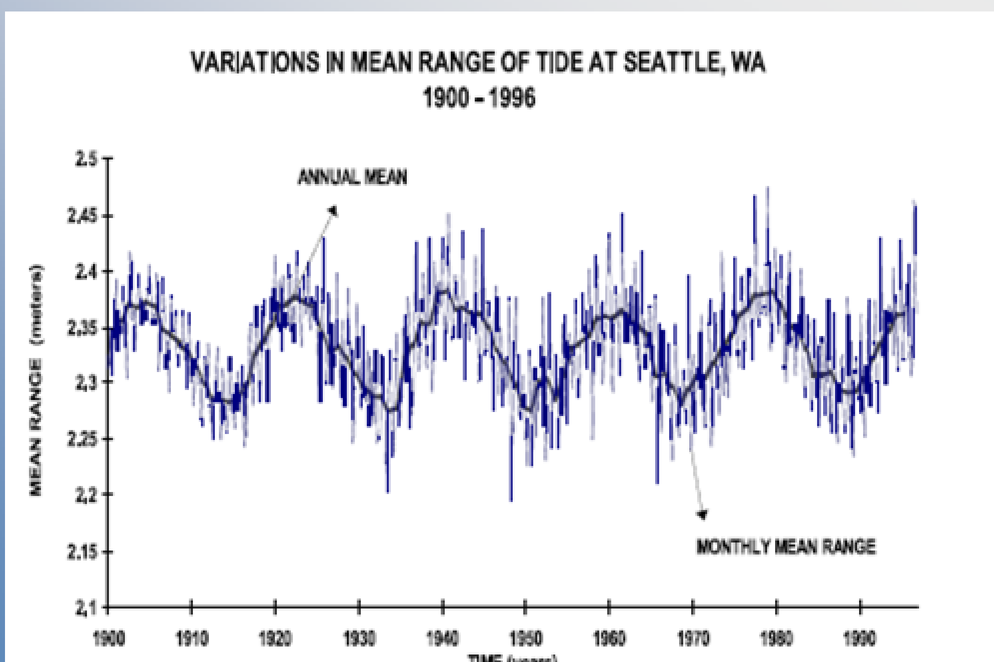
Plane of Earth's orbit around the sun (the ecliptic) is at an angle of ~23.5° to the equatorial plane

Summer solstice (June 21st) – the sun is furthest north of equator
Winter solstice (Dec. 21st) – the sun is furthest south of equator
Vernal equinox (Mar 21st) – the sun is over the equator
Autumnal equinox (Sep 21st) – the sun is over the equator

This leads to a 6-month modulation of the solar tide

Changing Orbital Parameters





Angle I is angle between the plane of moon's orbit and Earth's equatorial plane

- I varies of an 18.6-yr period between 28.5° to 18.5°)
- Cycle called the *lunar nodal regression*
  - Has a ±4% effect on M_2 and a ±11% on diurnal frequencies

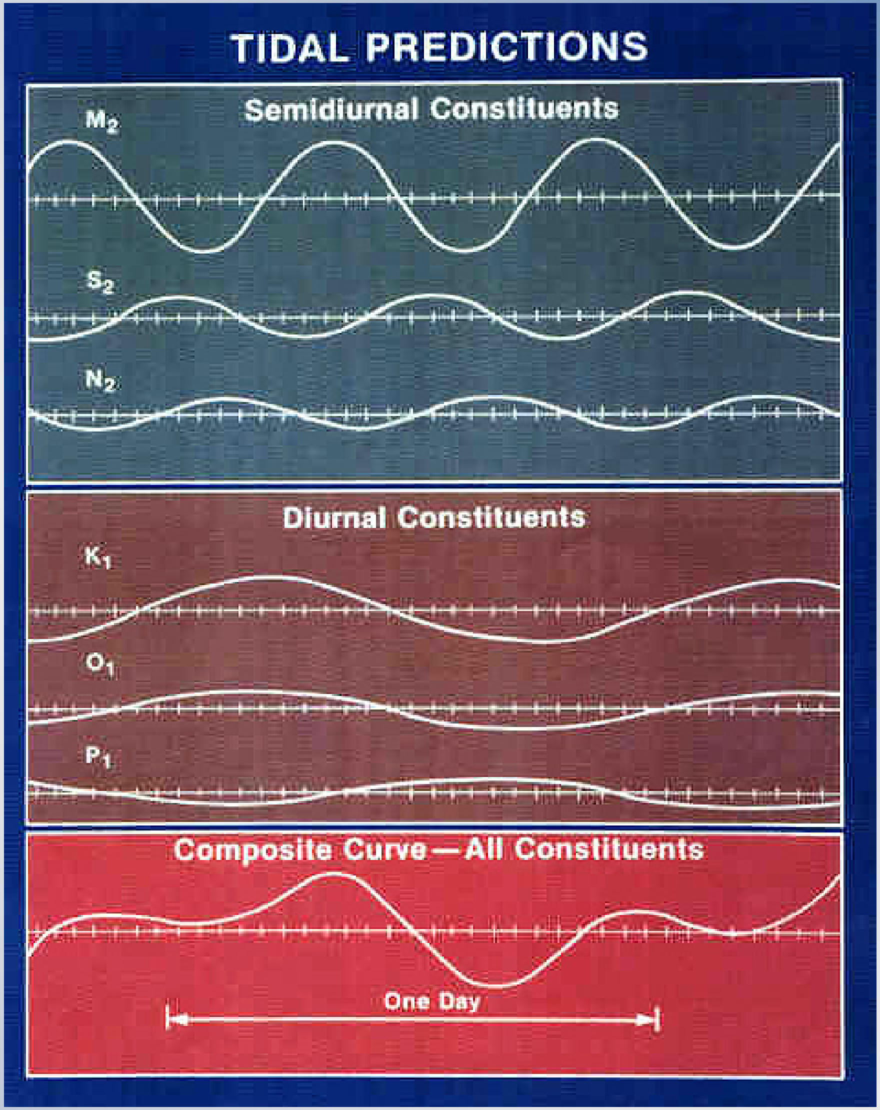
← Effect of the lunar nodal regression is clearly seen in the long-term water level series

Harmonic Analysis and Tide Predictions

- A prediction of the tide is based upon determining the contribution of the known tidal constituents (astronomical cycles) of the Earth-Moon-Sun system

- A sufficient time series of water level is decomposed into a sum of harmonic terms (cosine waves) of known frequency (or period) that will have different amplitudes and phases from the forcing

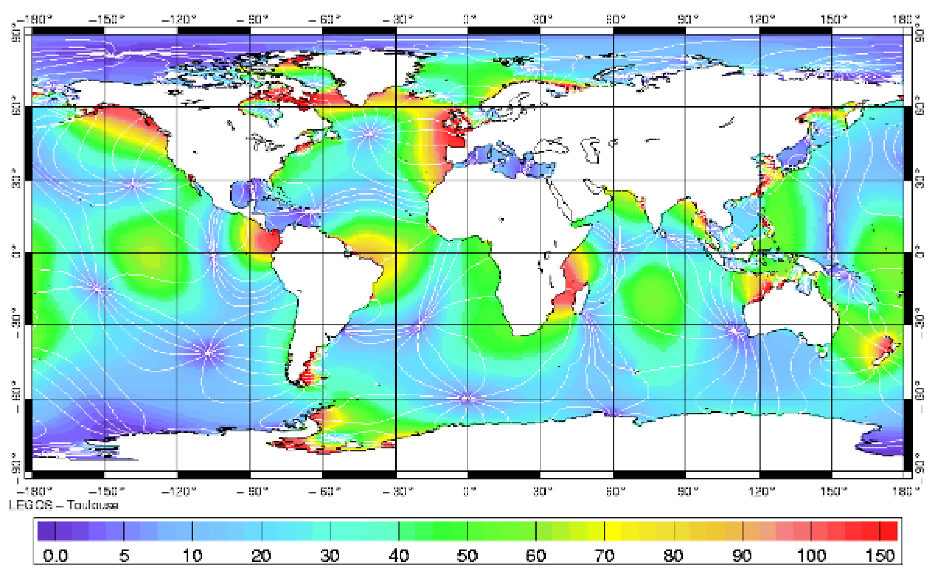
- NOAA CO-OPS generally uses a minimum of one year of hourly water level observations to compute the tide component frequencies to create predictions



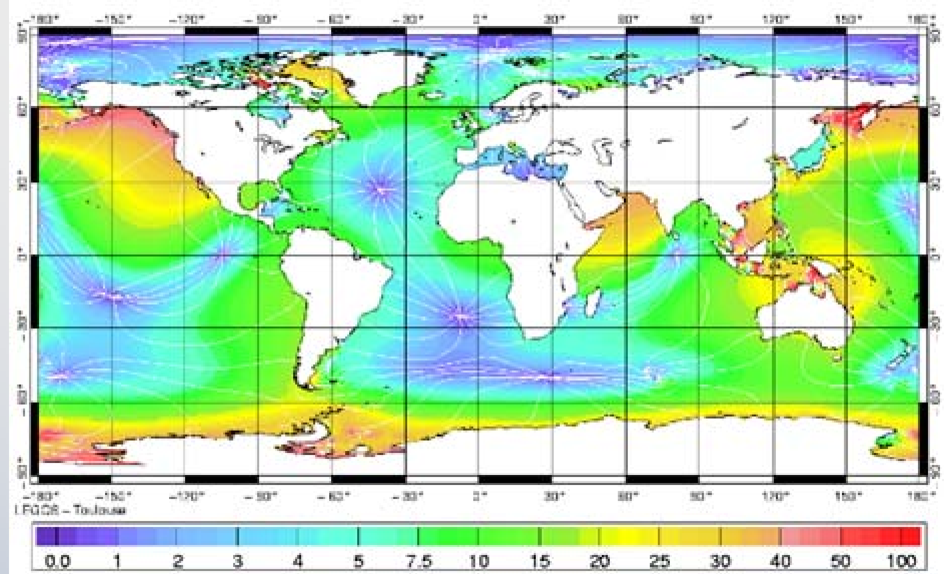
Hydrodynamics Modification of Tide Response

- Determines the range and timing of the tide and the observed high and low waters - Friction, Coriolis, wave reflections and width / depth changes
- In shallow waterways there is interaction with non-tidal phenomena: river flow and storm surges
- Basins have natural periods of oscillation or resonance that influence their tide characteristics
  - Pacific and the Gulf of Mexico are more resonant to diurnal components of the tide
  - Atlantic basin is more resonant to the semidiurnal tidal components

M2 (dominant semi-diurnal) constituent

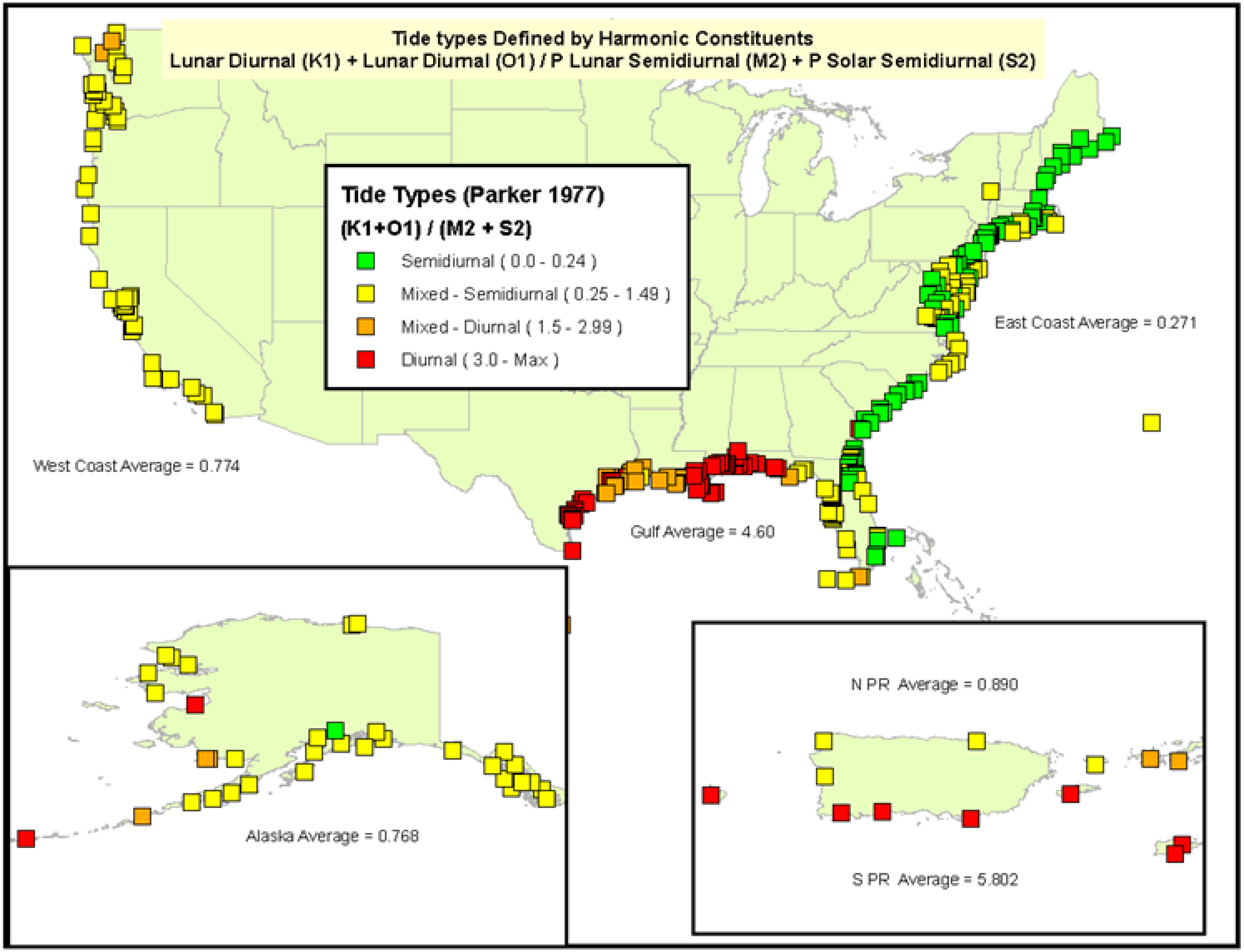


K1 (dominant diurnal) constituent

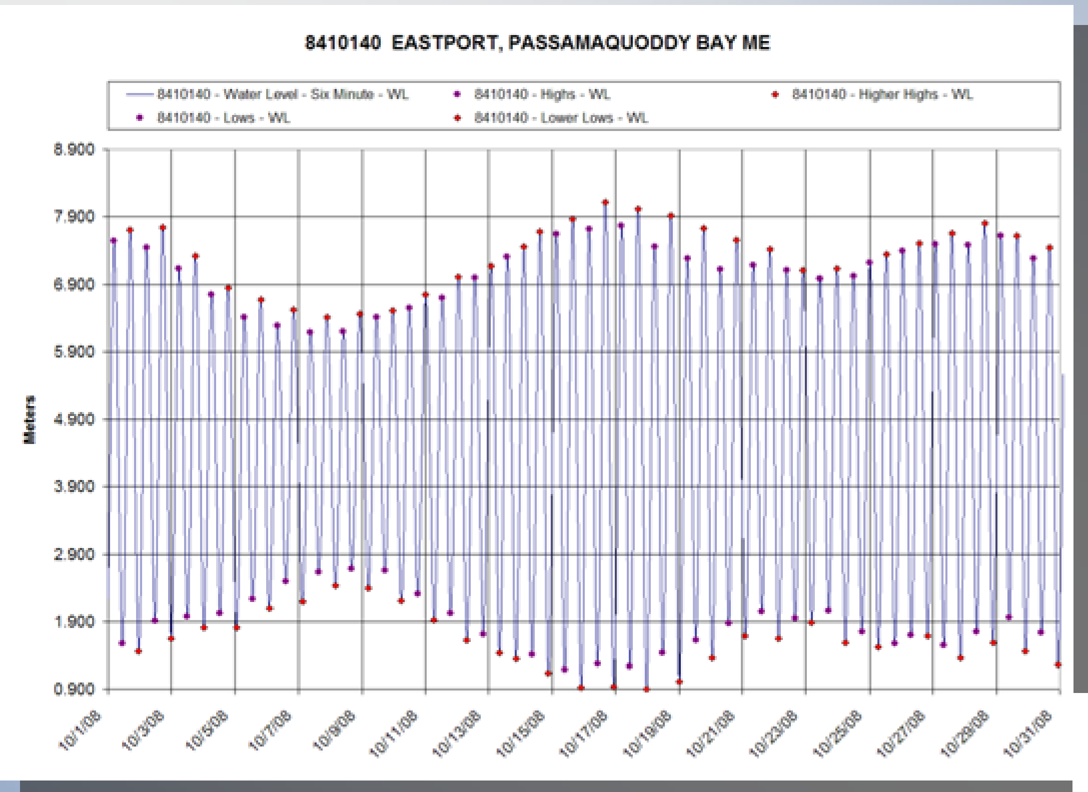


Amplitude of M2 and K1 tidal constituents (cm). White lines are co-tidal lines indicating phase every 30 degrees originate at amphidromic points where the tidal range is zero.

Classification of Tide Types at NOAA Water Level Stations

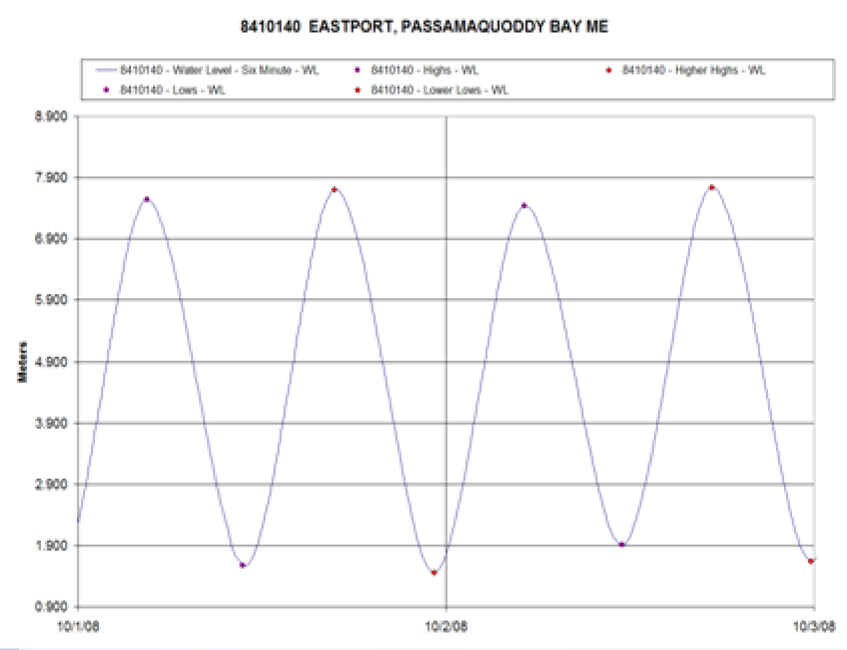


Classification of Tide Types at NOAA Water Level Stations

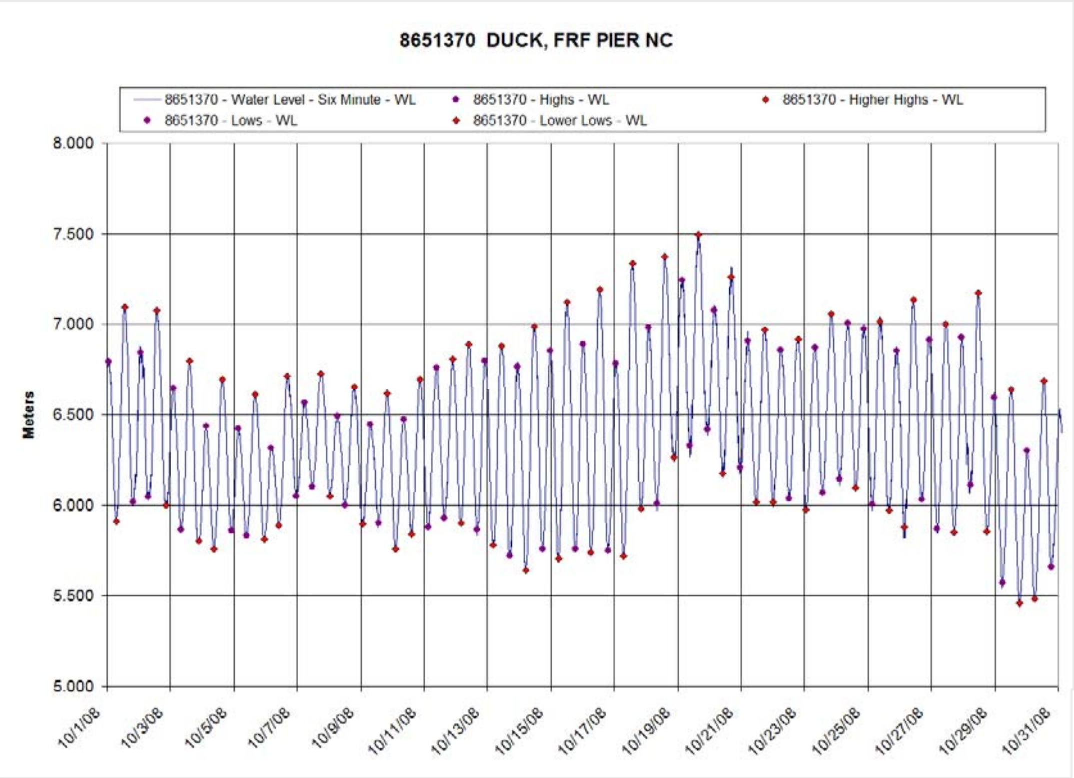


Semidiurnal signal

Eastport, Maine
(K1 + O1) / (M2 + S2) = 0.09



Classification of Tide Types at NOAA Water Level Stations

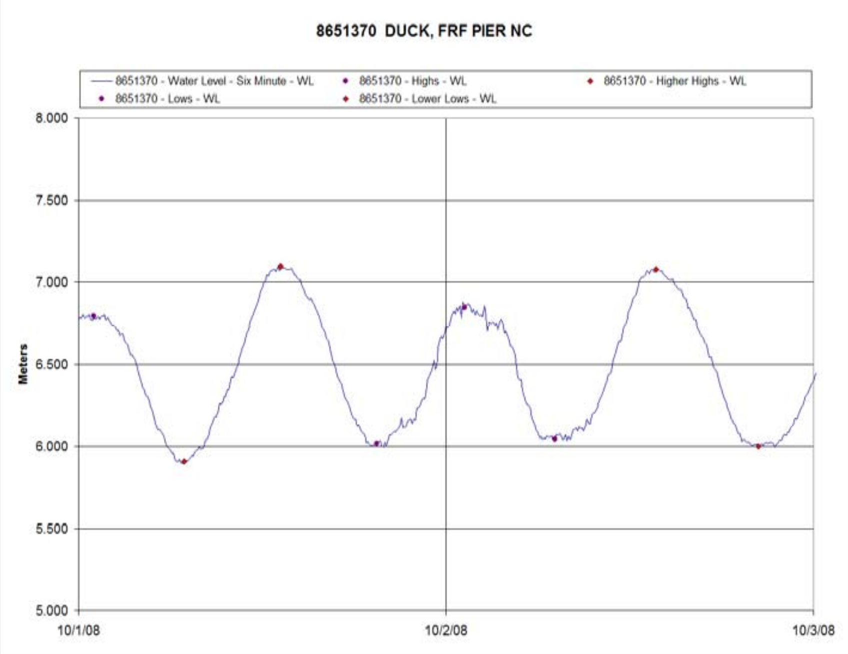


Transition between

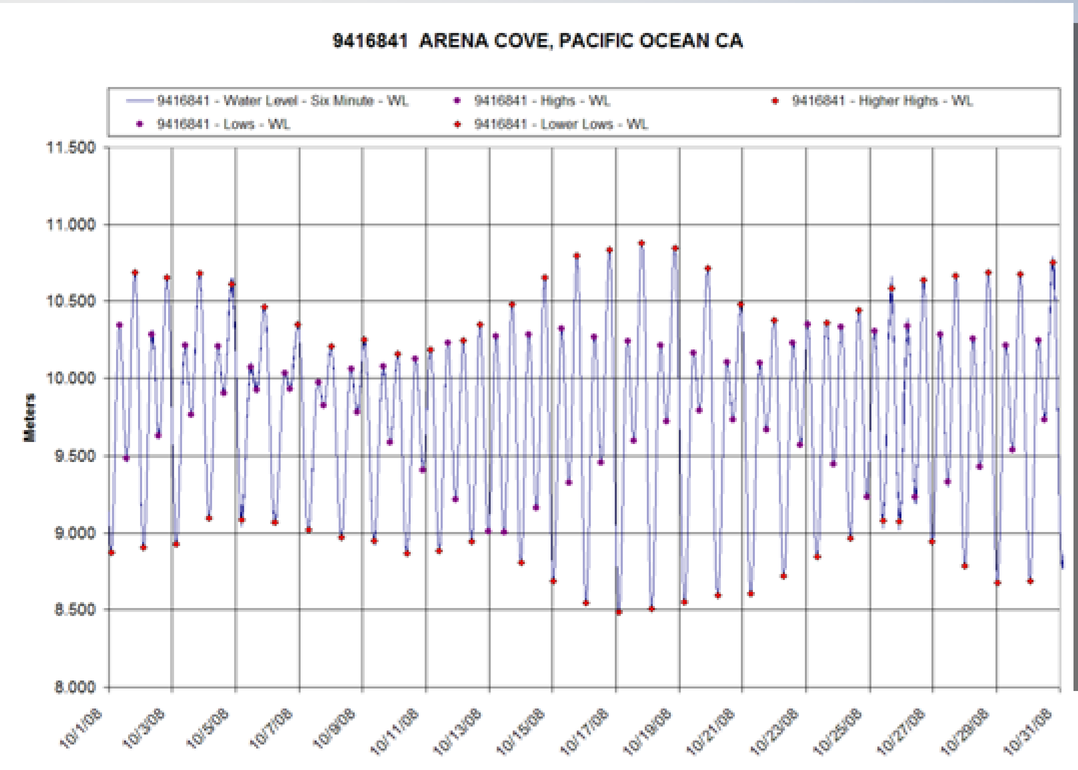
Semidiurnal and *Mixed-Semidiurnal* signals

Duck, North Carolina

(K1 + O1) / (M2 + S2) = 0.25



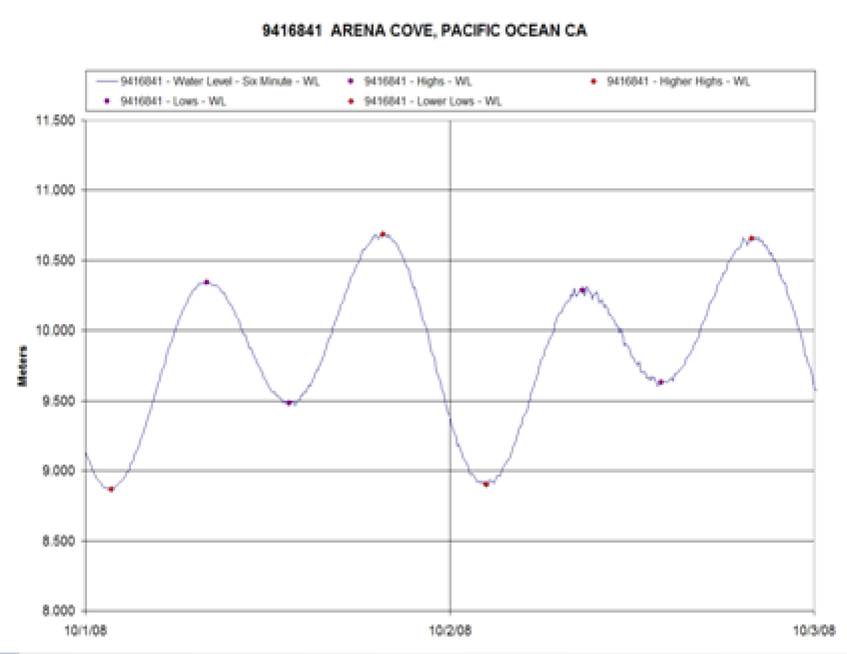
Classification of Tide Types at NOAA Water Level Stations



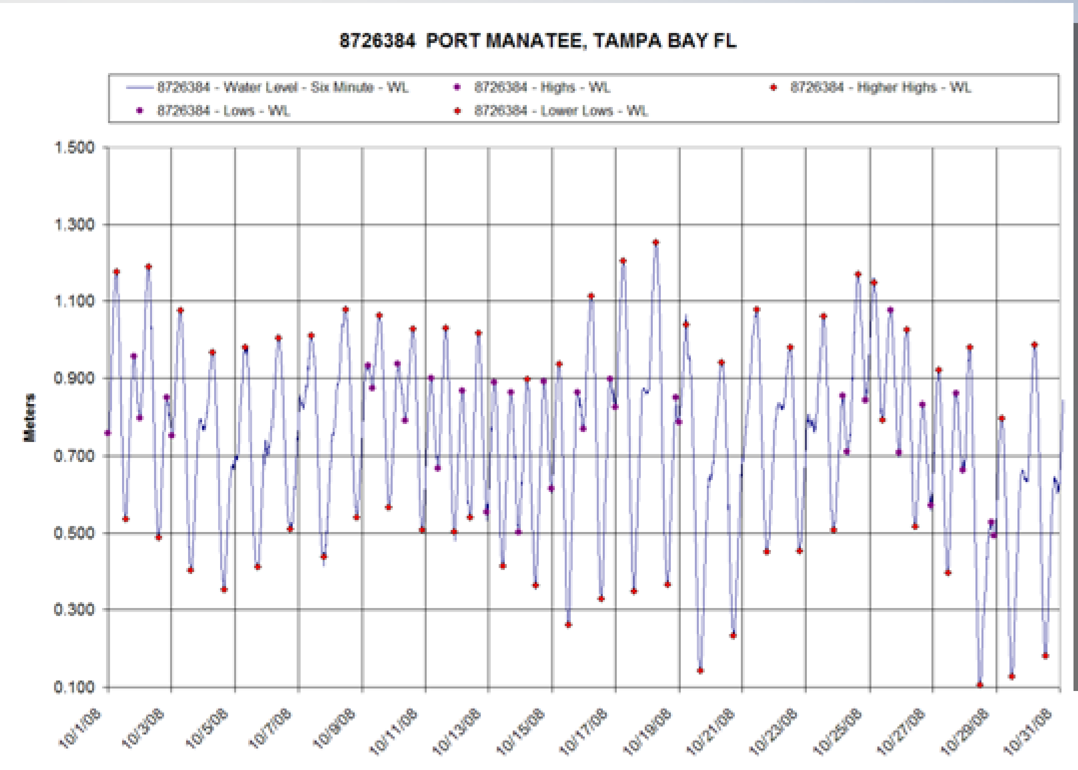
Mixed-Semidiurnal signal

Arena Cove, California

(K1 + O1) / (M2 + S2) = 0.85



Classification of Tide Types at NOAA Water Level Stations

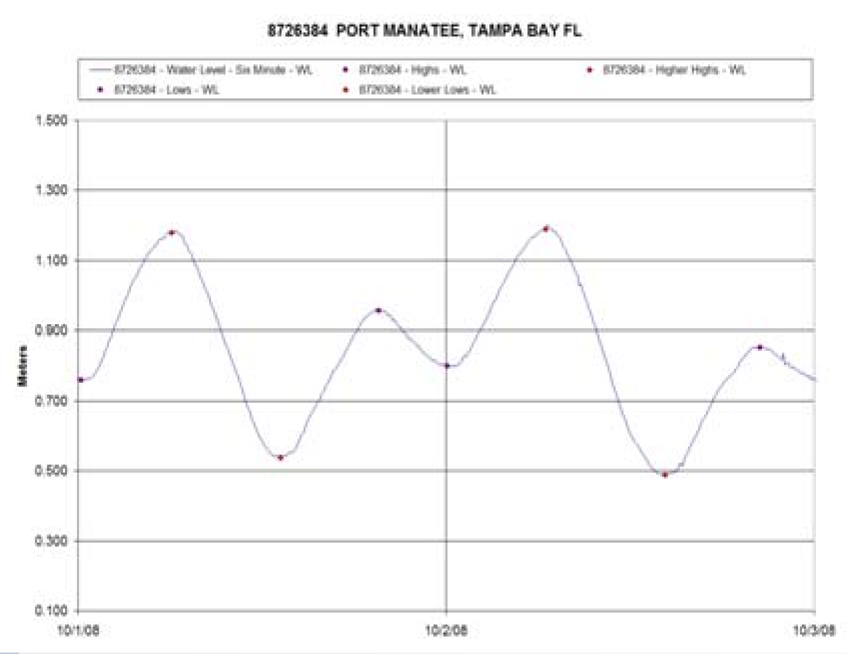


Transition between

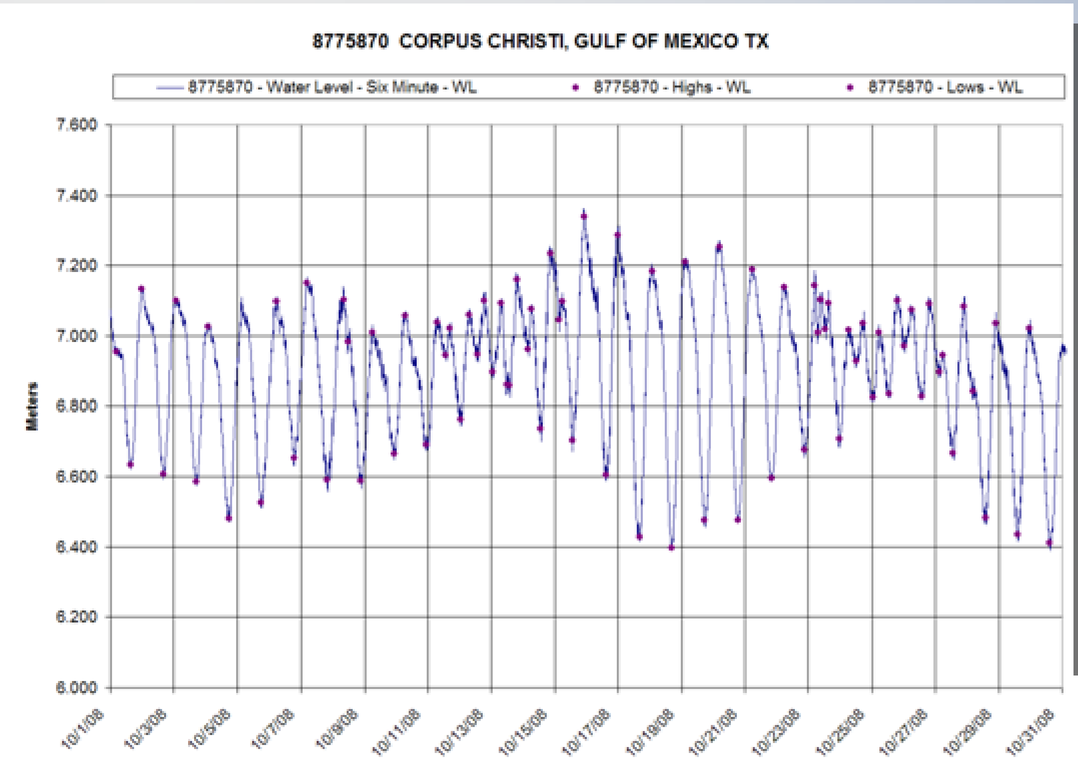
Mixed-Semidiurnal and *Mixed-Diurnal* signals

Port Manatee, Florida

(K1 + O1) / (M2 + S2) = 1.43



Classification of Tide Types at NOAA Water Level Stations

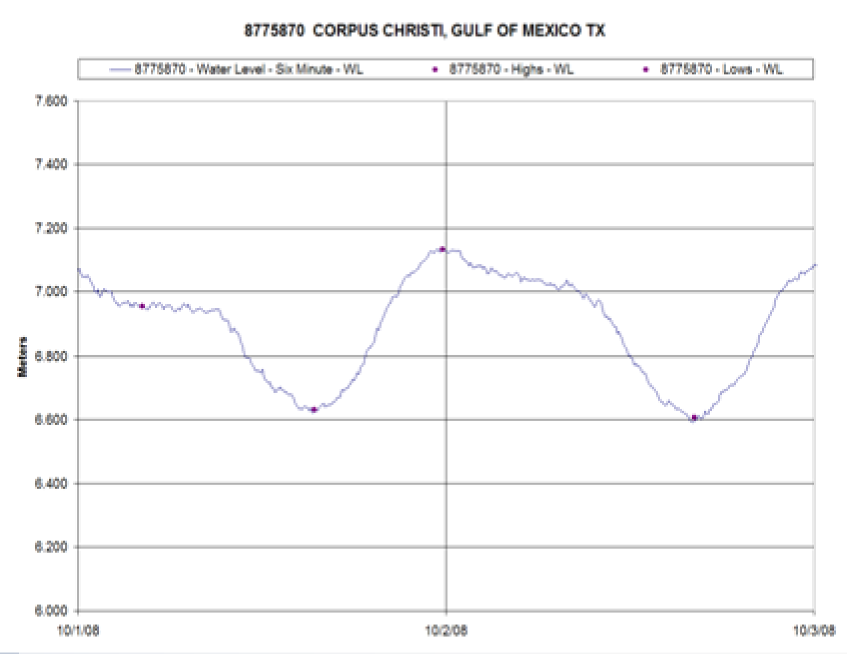


Transition between

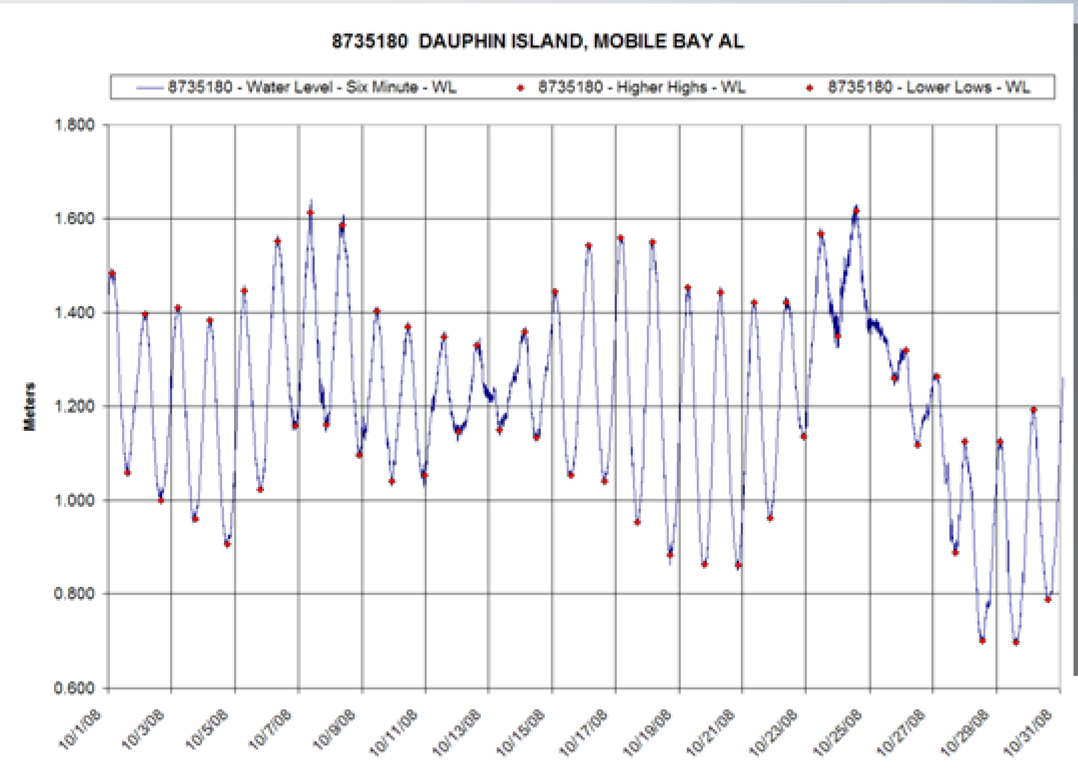
Mixed-Diurnal and *Diurnal* signals

Corpus Christi, Texas

(K1 + O1) / (M2 + S2) = 3.07

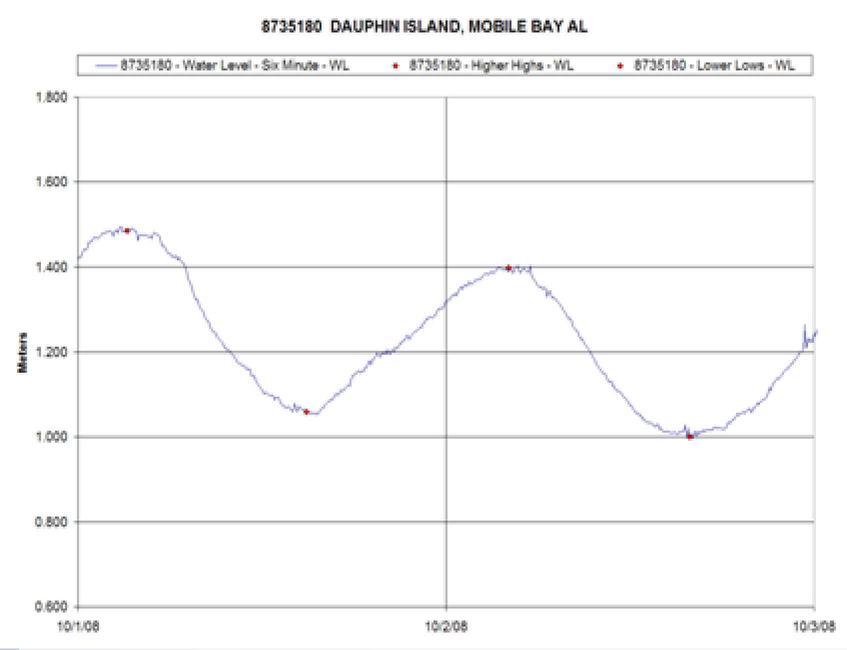


Classification of Tide Types at NOAA Water Level Stations



Diurnal signal

Dauphin Island, Alabama
(K1 + O1) / (M2 + S2) = 12.68



Tidal Theory
Tide Producing Forces

Stephen Gill
Stephen.Gill@noaa.gov